Identity Equations

Type One

- Multiply out both sides of the equation.
- Equate matching coefficients.
- You may need to solve a set of simultaneous equations.

Type Two

- Solve one equation.
- The coefficients of the next equation will be equal to one just solved.
- Put the solutions of the last equation equal to the new unknown expression.

Quadratic Equation Facts

Roots of a Quadratic Equation :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and Product of a Quadratic Equation :

A quadratic equation takes the form $ax^2 + bx + c = 0$ $\left(\text{i.e. } x^2 + \frac{b}{a}x + \frac{c}{a} = 0\right)$ where the:

sum of the roots: $\alpha + \beta = -\frac{b}{a}$

product of the roots: $\alpha\beta = \frac{c}{a}$

Identities to be familiar with :

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$

Forming an equation :

$x^2 - (\text{sum of roots})x + \text{product of the roots} = 0$

Nature of the roots of a Quadratic Equation :

$b^2 - 4ac \geq$ for real roots

$b^2 - 4ac < 0$ for unreal/complex roots

$b^2 - 4ac = 0$ for equal roots

Roots of a Quadratic

Solve $x^2 + 3x + 2 = 0$

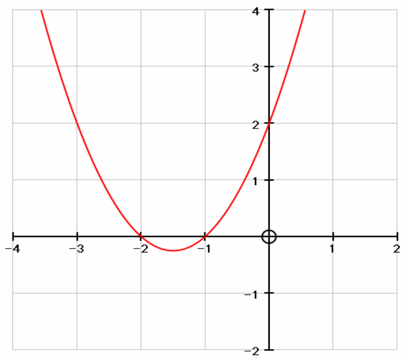
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{1}}{2}$$

$x = -2$ and $x = -1$

as $b^2 - 4ac > 0$

we have real distinct roots.



Solve $x^2 + 4x + 4 = 0$

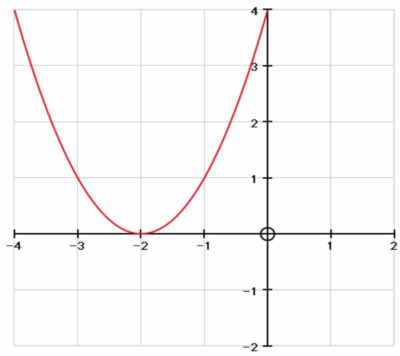
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{0}}{2}$$

$x = -2$ and $x = -2$

as $b^2 - 4ac = 0$

we have real equal roots.



Solve $x^2 + 6x + 13 = 0$

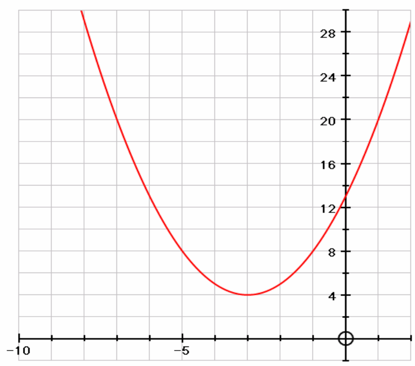
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{-16}}{2}$$

$\sqrt{-16}$ is not real

as $b^2 - 4ac < 0$

we have no real roots.



Related Roots

If there is a relationship between the roots given in the question, then instead of using α and β, express both of the roots in terms of α. (or β!)

The roots of the equation are consectutive integers. The roots of the equations differ by one.	α and $\alpha + 1$ or α and $\alpha - 1$
One root of the equation is twice the other.	α and 2α
The roots of the equation differ by two.	α and $\alpha + 2$ or α and $\alpha - 2$